Exercise 44

A particle moves along a straight line with equation of motion $s = f(t)$, where s is measured in meters and t in seconds. Find the velocity and the speed when $t = 4$.

$$f(t) = 10 + \frac{45}{t+1}$$

Solution

The velocity is the derivative of $s = f(t)$.

$$\begin{aligned}
f'(t) &= \lim_{h\to 0} \frac{f(t+h) - f(t)}{h} \\
&= \lim_{h\to 0} \frac{\left[10 + \frac{45}{(t+h)+1}\right] - \left[10 + \frac{45}{t+1}\right]}{h} \\
&= \lim_{h\to 0} \frac{\frac{45}{t+h+1} - \frac{45}{t+1}}{h} \\
&= \lim_{h\to 0} \frac{\frac{45(t+1)}{(t+h+1)(t+1)} - \frac{45(t+h+1)}{(t+1)(t+h+1)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{45(t+1)-45(t+h+1)}{(t+h+1)(t+1)}}{h} \\
&= \lim_{h\to 0} \frac{45(t+1) - 45(t+h+1)}{h(t+h+1)(t+1)} \\
&= \lim_{h\to 0} \frac{-45h}{h(t+h+1)(t+1)} \\
&= \lim_{h\to 0} \frac{-45}{(t+h+1)(t+1)} \\
&= \frac{-45}{(t+1)(t+1)} \\
&= -\frac{45}{(t+1)^2}
\end{aligned}$$

Therefore, the velocity when $t = 4$ is

$$f'(4) = -\frac{45}{(4+1)^2} = -1.8\ \frac{\text{m}}{\text{s}},$$

and the speed when $t = 4$ is

$$|f'(4)| = |-1.8| = 1.8\ \frac{\text{m}}{\text{s}}.$$